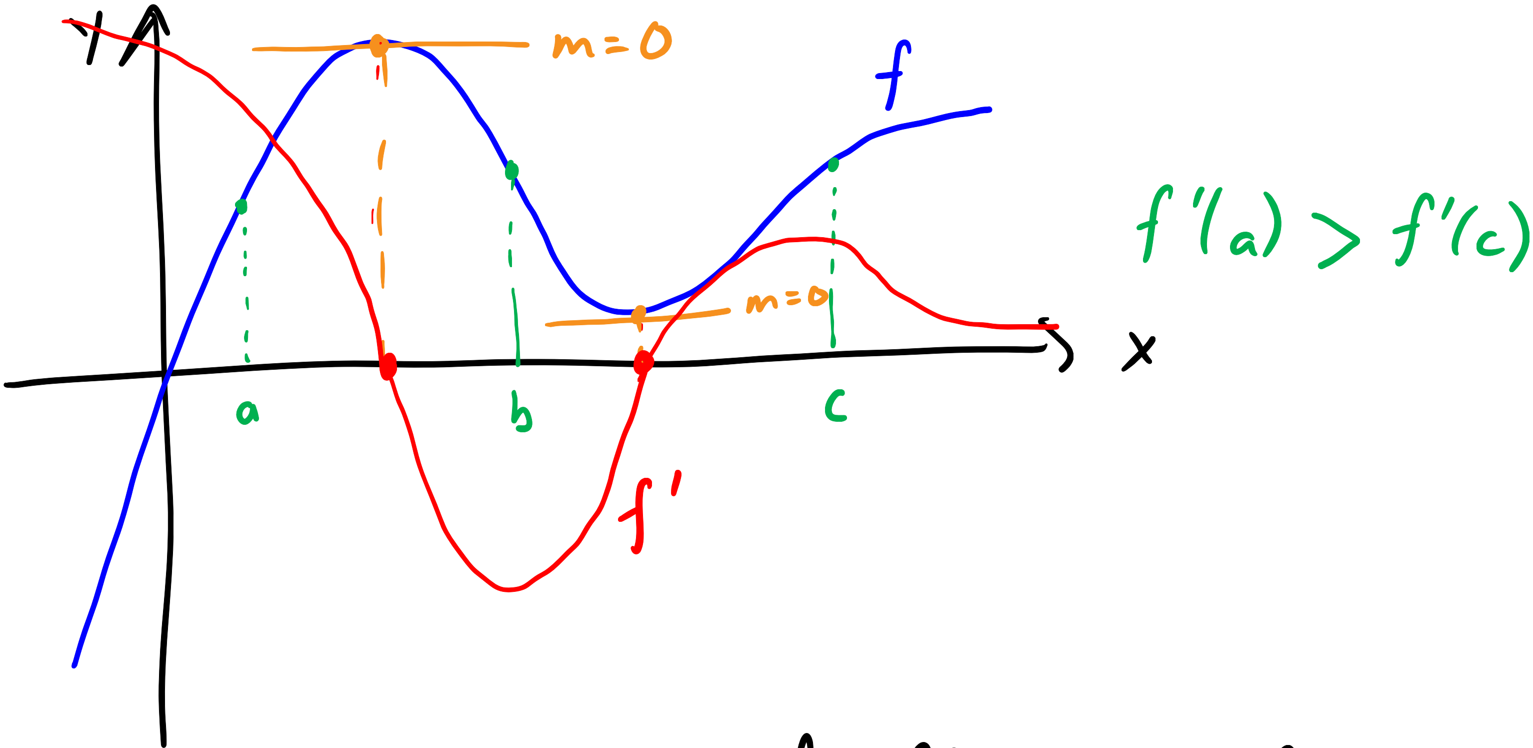


**Def**: The <u>derivative</u> of $f(x)$ is the function $f'(x)$ given by

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

provided the limit exists.

We can also write

$$\boxed{f'(x) = \lim_{t \to x} \frac{f(t) - f(x)}{t - x}}$$

Ex:

① $f(x) = 3x + 2$

$$f'(x) = \lim_{h \to 0} \frac{(3(x+h)+2) - (3x+2)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3x} + 3h + \cancel{2} - \cancel{3x} - \cancel{2}}{h} = \lim_{h \to 0} \frac{3\cancel{h}}{\cancel{h}}$$

$$= \lim_{h \to 0} 3 = 3 \qquad \boxed{f'(x) = 3}$$

② $f(x) = x^3$

$$f'(x) = \lim_{t \to x} \frac{t^3 - x^3}{t - x} = \lim_{t \to x} \frac{\cancel{(t-x)}(t^2 + tx + x^2)}{\cancel{t-x}}$$

$$= \lim_{t \to x} (t^2 + tx + x^2) = x^2 + x^2 + x^2 = 3x^2$$

③ $g(x) = \sqrt{9-x}$

$$g'(x) = \lim_{t \to x} \frac{\sqrt{9-t} - \sqrt{9-x}}{t - x} \cdot \frac{\sqrt{9-t} + \sqrt{9-x}}{\sqrt{9-t} + \sqrt{9-x}}$$

$$= \lim_{t \to x} \frac{(\cancel{9} - t) - (\cancel{9} - x)}{(t-x)(\sqrt{9-t} + \sqrt{9-x})} = \lim_{t \to x} \frac{-t + x}{(t-x)(\sqrt{9-t} + \sqrt{9-x})}$$

$$= \lim_{t \to x} \frac{-\cancel{(t-x)}}{\cancel{(t-x)}(\sqrt{9-t}+\sqrt{9-x})} = \lim_{t \to x} \frac{-1}{\sqrt{9-t}+\sqrt{9-x}}$$

$$= \frac{-1}{\sqrt{9-x}+\sqrt{9-x}} = \boxed{\frac{-1}{2\sqrt{9-x}} = g'(x)}$$

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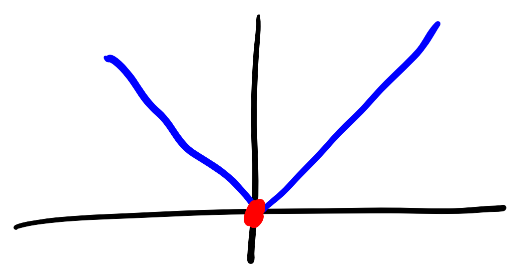
<u>Ex</u>: $f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$



When $x > 0$, $f'(x) = 1$

When $x < 0$, $f'(x) = -1$

To compute $f'(0)$:

$$\lim_{x \to 0^+} \frac{f(x)-f(0)}{x-0} = \lim_{x \to 0^+} \frac{x-0}{x-0} = \lim_{x \to 0^+} 1 = 1$$

$$\lim_{x \to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{x \to 0^-} \frac{-x-0}{x-0} = \lim_{x \to 0^-} -1 = -1$$

So, $f'(0) = \lim_{x \to 0} \frac{f(x)-f(0)}{x-0}$ DNE

So $f(x) = |x|$ is not differentiable at $x=0$.

**Def**: A function $f$ is <u>differentiable at $a$</u> if $f'(a)$ exists. It is <u>differentiable on an open interval</u> $(a,b)$ (or $(-\infty, b)$ or $(a,\infty)$ or $(-\infty,\infty)$) if it is differentiable at every number in the interval.

<u>3 ways to not be differentiable</u>

① $f$ is not continuous

② $f$ has a corner ($|x|$)

③ $f$ has a vertical tangent ($\sqrt[3]{x}$)

(would need $f' = \pm\infty$)